The Effect of Meeting Rates on Matching Outcomes*

Sonia Jaffe   Simon Weber

March 7, 2018

Abstract

We extend the classic matching model of Choo and Siow (2006) to allow for the possibility that the rate at which potential partners meet affects their probability of matching. We investigate the implications for the levels and supermodularity of the estimated match surplus. **JEL Codes:** C78 · J12

*The authors are grateful for the insightful comments of Pierre-Andre Chiappori, Alfred Galichon, Scott Kominers, and workshop participants at the University of Chicago.

1 Introduction

Many matching markets – marriage markets, student-to-school matching, etc – have been analyzed using models in the style of Choo and Siow (2006). These models allow for some unobserved heterogeneity and account for how changes in the number of agents of each type affect matching patterns. However, all agents on the other side of the market are assumed to be potential match partners – agents do not *meet* (and therefore cannot meet at different rates). We extend the Choo and Siow (2006) standard matching framework to incorporate the idea of agents meeting; this changes the relationship between match surplus and matching patterns. The standard Choo and Siow (2006) framework overestimates the systematic match surplus for groups with many individuals because it does not account for the fact that a given individual will meet more people from a more populous group.

We also consider the effect of requiring potential partners to meet on the assortativeness of matching – the extent to which agents of the same type match together. One type of assortativeness where meeting is potentially important is educational assortativeness in marriage markets, since it is reasonable to think that many people meet potential spouses at school or work where people of their education level are disproportionately represented. One can measure assortativeness by computing the supermodularity of the surplus in the Choo and Siow (2006) model. Such measures of assortativeness combine preferences with the effects of differential availability – different types meeting at different rates.[1] Our framework gives an alternative measure of supermodularity, adjusted for meeting rates. Differentiating between preferences and availability is relevant both for estimating the value of different types of matches – e.g. for understanding household production – and for the efficiency of the market. Assortativeness due to preferences is efficient whereas assortativeness due to differential meeting rates results from the friction of needing to meet potential partners.[2]

Many papers have shown an increase in the educational assortativeness of marriage in the United States over the second half of the twentieth century (e.g. **?**Eika et al, 2017);[3] one hypothesis to explain the trend is changes in 'meeting frequencies' due to population shifts – e.g. more women are college-educated.[4] However, we show that a simple effect where individuals are x times as likely to meet an individual of their own type *does not* generate changes in assortativeness when relative populations of types shift, so it cannot explain the observed changes in marriage patterns. We consider an alternative model where individuals of a given type (e.g. education level) meet some people (e.g. at work) only from their own group and meet others (e.g. at bars) in proportion to their numbers in the population; importantly, the more potential partners they meet at work, the fewer they seek to meet in the general population. In this model, changes in relative populations can generate changes

[1] There is, however, evidence that individuals have at least some preference for partners with similar education levels, e.g. in Bruze (2011) and Belot and Francesconi (2013).

[2] As we discuss in Section 3, the outcome is still constrained-efficient.

[3] An exception is Gihleb and Lang (2016). Papers in sociology and demography (e.g., Schwartz and Mare, 2005; Kalmijn, 1991) have also measured assortativeness and analyze its trends using statistical methods such as log-linear models. Some recent work in economics has focused on the impact on between-household inequalities (Greenwood et al, 2014).

[4] Most data on marriages do not include how the couple met, but retrospective survey evidence suggests that the fraction of heterosexual couples that meet in a given year through college increased between 1940 and 2000 (Rosenfeld and Thomas, 2012).

in the assortativeness of the match. Our paper relates to the literature on matching with perfectly transferable utility. We consider equilibrium matchings, in the spirit of Shapley and Shubik (1971), extending the classic Choo and Siow (2006) model. Our concept of agents 'meeting' is much more reduced form than the dynamic models of the search and matching literature, where agents receive match offers at a given rate (e.g. Lu and McAfee (1996) and Shimer and Smith (2000) ; see Goussé et al (2017) for a state-of-the-art application to the marriage market). We believe our model is better suited to the cross-sectional data typically used to estimate models in the spirit of Choo and Siow. Some data (or assumptions) on the average number of meetings is needed, but not the rate or timing of meetings or offers.

Our model is also closely related to Dupuy and Galichon (2014); in their continuous version of the Choo and Siow setting, men (resp. women) only have access to a set of acquaintances, which is a random subset of the whole population of women (resp. men). An extension in Menzel (2015), which considers a non-transferable utility matching model also allows for the idea of meeting a subset of agents. However, neither of these papers allow the probability of meeting to vary with type or consider how changes in meeting rates affect match patterns.

Organization of the paper. Section 2 introduces our extended Choo and Siow framework. In Section 3, we detail how surplus is computed and assortativeness is measured. Section 4 considers various parameterization of meeting rates. Finally, Section 5 concludes.

2 Model

We present two ways for thinking about how differences in the prevalence of meetings between types may affect matching patterns.

Sub-types

The mechanics of Choo and Siow (2006) (CS) require a mass of agents of each potential partner types, so instead of thinking of agents meeting individuals, we model agents as meeting partners of unobservably different subtypes. This allows us to use the number of subtypes of a given type that an agent meets as a measure of the number of meetings – the 'meeting rate' – between that agent and that type.

In our marriage market, a man k and a woman ℓ are categorized by an observable type $x \in \mathcal{X}$ and $y \in \mathcal{Y}$, respectively. Additionally, each type is divided into equal-sized unobservable sub-types $i_k \in \mathcal{I}$ and $j_\ell \in \mathcal{J}$. Types may vary by size; we assume that the mass of each subtypes is fixed to 1, so differences in type populations is reflected in differences in the number of sub-types. There are n_x subtypes of men within type x (i.e. $|\{i|x_i = x\}|$) and m_y subtypes of women within y, where n_x and m_y are finite for all x, y.

Each subtype of men only meets women from some subtypes and a man cannot match with a woman he does not meet.[5] The set of subtypes of women that men in subtype i meet make up their choice set $C^i = \cup_{y \in \mathcal{Y}} c^i_y$, where c^i_y is a random subset of $\{j|y_j = y\}$. We assume

[5]Equivalently, assume that the idiosyncratic part of the match is infinitely negative for women of other subtypes.

that meeting is reciprocal so that $i \in C^j = \cup_{x \in \mathcal{X}} c_x^j$ if and only if $j \in C^i$. The number of meetings varies only by type so $|c_y^i| = a_{x_i y}$. The number of subtypes of x that a woman of type y meets, b_{xy}, is constrained by an adding up constraint: $m_y b_{xy} = a_{xy} n_x$.

Each man of subtype i must choose whether to remain single or match with a woman of one of the $a_{x_i} = \sum_{y \in \mathcal{Y}} a_{x_i y}$ subtypes that he meets. The non-random component of preferences depends only on type so all sub-types within a type are ex-ante identical. They vary only in which other subtypes they meet and their random preference draws for those subtypes. When man k of subtype i matches with a woman of subtype j, his utility is

$$\alpha_{x_i y_j} - \tau_{x_i y_j} + \epsilon_{kj}$$

where $\alpha_{x_i y_j}$ is a systematic component of i's utility, $\tau_{x_i y_j}$ is an equilibrium transfer paid and ϵ_{kj} is a random component to utility that depends on the subtype of the partner. Similarly, if woman ℓ of subtype j chooses a man from subtype i from her $b_{y_j} = \sum_{x \in \mathcal{X}} b_{xy}$ options, her utility has a type-based systematic component, a transfer, and random component:

$$\gamma_{x_i y_j} + \tau_{x_i y_j} + \eta_{i\ell}.$$

Without loss of generality, we normalize the systematic component of being single to zero so if man k or woman ℓ chooses to remain single, they receive utilities ϵ_{k0} and $\eta_{0\ell}$, respectively. Finally, we assume that the random components of utility are independently drawn from a standard Extreme Value Type I distribution with a standard deviation of 1. We can now derive an aggregate matching function which generalizes the aggregate matching function from Choo and Siow (2006) to take into account meeting rates.

Proposition 1. *In the CS model with meeting rates, the aggregate matching function is*

$$\mu_{xy} = \sqrt{a_{xy} b_{xy}} \sqrt{\mu_{0y} \mu_{x0}} \exp\left(\frac{\phi_{xy}}{2}\right) \tag{1}$$

where $\phi_{xy} \equiv \alpha_{xy} + \gamma_{xy}$

Proof. The mass of matches between men i and women j if they meet is

$$\mu_{ij} = \frac{\exp(\alpha_{x_i y_j} - \tau_{x_i y_j})}{1 + \sum_{j' \in C^i} \exp(\alpha_{x_i y_{j'}} - \tau_{x_i y_{j'}})}, \qquad \mu_{ij} = \frac{\exp(\gamma_{x_i y_j} + \tau_{x_i y_j})}{1 + \sum_{i' \in C^j} \exp(\gamma_{x_{i'} y_j} + \tau_{x_{i'} y_j})}.$$

The mass of single men of subtype i and single women of subtype j are

$$\mu_{i0} = \frac{1}{1 + \sum_{j \in C^i} \exp(\alpha_{x_i y_j} - \tau_{x_i y_j})}, \qquad \mu_{0j} = \frac{1}{1 + \sum_{i \in C^j} \exp(\gamma_{x_i y_j} + \tau_{x_i y_j})}.$$

In Choo and Siow terms, these are supply and demand equations. Therefore, at equilibrium these give

$$\mu_{ij}^2 = \exp(\alpha_{x_i y_j} + \gamma_{x_i y_j}) \mu_{i0} \mu_{0j} \tag{2}$$

Since subtypes are ex-ante identical, we expect them to have the same probability of being single.[6] Therefore, the mass of single men of type x is $\mu_{x0} = \mu_{i0} n_x$ for all $i : x_i = x$ (and similarly for women). The mass of matches between a man i women of type y is

$$\mu_{iy} = \sum_{j \in c_y^i} \mu_{ij} = a_{x_i y} \left(\exp(\alpha_{x_i y_j} + \gamma_{x_i y_j}) \frac{\mu_{0y}}{m_y} \frac{\mu_{x0}}{n_x} \right)^{\frac{1}{2}}.$$

The expected number of matches between men of type x and woman of type y is

$$\begin{aligned} \mu_{xy} = \sum_{i | x_i = x} \mu_{iy} &= n_x a_{x_i y} \left(\exp(\alpha_{x_i y_j} + \gamma_{x_i y_j}) \frac{\mu_{0y}}{m_y} \frac{\mu_{x0}}{n_x} \right)^{\frac{1}{2}} \\ &= \left(a_{xy} b_{xy} \exp(\alpha_{x_i y_j} + \gamma_{x_i y_j}) \mu_{0y} \mu_{x0} \right)^{\frac{1}{2}}, \end{aligned}$$

where the second line follows from the adding up constraint $n_y b_{xy} = a_{xy} m_x$. □

Proposition 1 calls for two comments. First, we can recover the CS aggregate matching function from the aggregate matching function in (1), with $a_{xy} = b_{xy} = 1$. Since in the Choo and Siow model, women of a given type are identical from man i perspective, this means that this man meets only one "representative" woman of each type. Second, our model belongs to the class of aggregate matching function equilibrium models. In these models, equilibrium is fully characterized by the set of nonlinear equations

$$\begin{aligned} \mu_{x0} + \sum_y M_{xy}(\mu_{x0}, \mu_{0y}) &= n_x \\ \mu_{0y} + \sum_x M_{xy}(\mu_{x0}, \mu_{0y}) &= m_y \end{aligned}$$

where the matching function M_{xy} is, in our case,

$$M_{xy}(\mu_{0y}, \mu_{x0}) = \sqrt{a_{xy} b_{xy}} \sqrt{\mu_{0y} \mu_{x0}} \exp\left(\frac{\phi_{xy}}{2} \right).$$

Galichon et al (2017) show that existence and uniqueness of an equilibrium is obtained whenever the aggregate matching function is increasing in its arguments, continuous and tends to zero as $\mu_{x0} \to 0$ or $\mu_{0y} \to 0$. These sufficient conditions are met in our setting.

Familiarity

Instead of thinking of meeting as restricting the choice set, we can think of it as increasing an individual's preference for a certain type of person. This is similar in spirit to Mourifié

[6]Indeed, the matching μ_{ij} is given by the matching function that appears implicitly in (2) and must satisfy the constraints $\mu_{i0} + \sum_{C^i} \sqrt{\mu_{i0} \mu_{0j}} \exp(\frac{\alpha_{x_i y_j} + \gamma_{x_i y_j}}{2}) = 1$ for all subtypes i (the same constraints must hold on the woman side of the market). Since for any two subtypes i and i' of the same type x the mass of these subtypes, the number of meetings and the surplus with any women of given type y is assumed to be the same, it must be the case that $\mu_{i0} = \mu_{i'0}$.

and Siow (2015) who model peer effects by having the utility of a man of type x married with a woman of type y depends positively on the number of matches between those types in the population. In our model, we instead have that a man's utility of matching depends on the number of women of that type that he meets. One can reasonably think that knowing more women of a given type could make men enjoy their company more. The utility of man k of matching with a woman of type y when he meets $a_{x_k y}$ women of that type is

$$\alpha_{x_k y} - \tau_{x_k y} + \ln(a_{x_k y}) + \epsilon_{ky}.$$

For woman ℓ, the utility of matching with a man of type x is

$$\gamma_{x y_\ell} + \tau_{x y_\ell} + \ln(b_{x y_\ell}) + \eta_{x\ell}.$$

These give match probabilities

$$\mu_{xy} = \left(a_{xy} b_{xy} \exp(\alpha_{x_i y_j} + \gamma_{x_i y_j}) \mu_{0y} \mu_{x0}\right)^{\frac{1}{2}} \tag{1'}$$

which are the same as in the model based on sub-types.

Equivalence

Since the remainder of the analysis relies only on the equation for μ_{xy} found in Equations (1) and (1'), the two frameworks for conceptualizing the model have all of the same implications. For the sake of simplicity, we mostly stick to the language of the familiarity model for the rest of the paper.

3 Measuring the surplus and assortativeness

3.1 Surplus

Analyses of matching markets frequently try to estimate the systematic surplus of match-pairs of different types.

Proposition 2. *In the CS model with meeting rates,*

(i) the total systematic surplus when a man of type x and a woman of type y match is

$$\phi_{xy} = \log\left(\frac{\mu_{xy}^2}{\mu_{x0}\mu_{0y}}\right) - \log(a_{xy} b_{xy}).$$

(ii) the total systematic surplus relates to the CS surplus through

$$\phi_{xy}^{CS} = \log\left(\frac{\mu_{xy}^2}{\mu_{x0}\mu_{0y}}\right) = \phi_{xy} + \log(a_{xy} b_{xy}).$$

(iii) if the random errors are correlated within types (with ρ denoting this correlation, that is $E[\epsilon_{ij}\epsilon_{ik}] = \rho$ if $y_j = y_k$)[7]*, then*

$$\phi_{xy}^{CS} = \log\left(\frac{\mu_{xy}^2}{\mu_{x0}\mu_{0y}}\right) = \phi_{xy} + (1-\rho)\log(a_{xy}b_{xy}).$$

Proof. The proof of (i) follows immediately by inverting Equation (1). We deduce (ii) by definition of the surplus in the CS model. A detailed proof of (iii) can be found in Appendix A. □

It follows from Proposition 2 that the CS model will over estimate the surplus between groups that are very likely to meet each other because it attributes the high match rate to high surplus instead of to a higher rate of meeting. In addition, if we are thinking of a_{xy} and b_{xy} as the number of subtypes a person meets within a type, then correlation of random errors within a type decreases the usefulness and the effect of meeting additional subtypes of a given type. Note that perfect correlation ($\rho = 1$) corresponds to the CS case of only getting one random draw per type.

Overall welfare is the sum of the systematic surplus and the idiosyncratic, random component of utility. In the standard CS model, the equilibrium matching maximizes total welfare. In our context, the equilibrium matching maximizes utility for a given a_{xy} and b_{xy}. Increasing the number of meetings (a_{xy} or b_{xy}) increases welfare. From the perspective of subtypes, increasing a_{xy} or b_{xy} relaxes a constraint – since agents are constrained to match with a sub-type they have met. In the context of familiarity, increasing the number of meetings raises agents' utility from a given match, thereby increasing equilibrium welfare.

3.2 Assortativeness and Supermodularity

The assortativeness of a match is the extent to which agents match more with their own type than with other types. Assortativeness of the match is related to the supermodularity of the match surplus – how much more surplus is generated when two agents of type A match and two agents of type B match compared to having two matches where one agent is type A and one is type B. Letting $x = y = t$ and $x' = y' = t'$, with $t \neq t'$, the supermodularity across these two types is

$$SM_{t,t'} \equiv \phi_{t,t} + \phi_{t',t'} - \phi_{t,t'} - \phi_{t',t}.$$

The different meeting rates make the supermodularity corresponding to a given matching pattern different from the standard CS model. If SM^{CS} is the Chow and Siow measure of supermodularity, the supermodularity adjusted for meeting rates is

$$SM_{t,t'} = \log\left(\frac{\mu_{tt}\mu_{t't'}}{\mu_{t't}\mu_{tt'}}\right) - \log\left(\frac{a_{t't'}b_{t't'}}{a_{t't}b_{t't}}\frac{a_{tt}b_{tt}}{a_{tt'}b_{tt'}}\right) = SM_{t,t'}^{CS} - \log\left(\frac{a_{t't'}b_{t't'}}{a_{t't}b_{t't}}\frac{a_{tt}b_{tt}}{a_{tt'}b_{tt'}}\right). \quad (3)$$

If people are more likely to meet others of their own type, then CS over-estimates the supermodularity of the matching surplus.

[7] Since ϵ is mean zero and variance 1, $\rho = \frac{E[\epsilon_{ij}-\mu_{\epsilon_{ij}}][\epsilon_{jk}-\mu_{\epsilon_{jk}}]}{\sigma_{\epsilon_{ik}}\sigma_{\epsilon_{jk}}} = E[\epsilon_{ij}\epsilon_{ik}]$.

3.3 Measurement

The meeting frequencies cannot be non-parametrically identified separately from the match surplus using only data on matching patterns. However, with logical parameterizations of the frequencies, one can look at how meeting patterns affect the matching and the estimated surplus.

Note that since we have normalized the standard deviation of the random errors to one, the surplus is measured in standard deviation units. If the surplus of matching with a college educated partner is .5 higher than with a high school educated partner, that means it is equivalent to moving up .5 standard deviations in the within-type distribution of match utilities. If errors are uncorrelated, increasing the number of meetings between two types of agents by 1% while decreasing their match surplus by .01 standard deviations (of the within-type distribution of match utilities) would have no effect.

4 Paramaterizing the meeting rates

We consider three models for how agents meet potential partners and discuss their implications for matching patterns and surplus measurement.

4.1 Baseline: Random meeting

If meetings are completely random, an individual meets people of each type in proportion to their numbers in the population.

$$a_{xy} = a_x \frac{m_y}{M} \qquad\qquad b_{xy} = b_y \frac{n_x}{N}. \tag{4}$$

The adding up constraints ($a_{xy} \cdot n_x = b_{xy} \cdot m_y$) for each pair of types require that $a_x \cdot N = b_y \cdot M$ for every x and y, so the number of potential partners an agent meets cannot vary by type, $a_x = a \; \forall x$ and $b_y = b \; \forall y$ with $\frac{a}{b} = \frac{M}{N}$.[8]

In this case, a model that did not account for the number of meetings between types would over estimate the surplus of matching with someone from a large group because it would not take into account that individuals were meeting more people from that group. (If an individual's preference does not vary across members of a group, it matters less how many of them she meets.) The meeting rate a and the average surplus of matching are not separately identified in a single market, but allowing for different meeting rates still matters. Adjusting for changes in the populations m_y and n_x over time affects estimates of how the surplus for different groups has changed. However, adjusting for meeting rates will not affect estimates of how the supermodularity of the surplus has changed over time.

Proposition 3. *In the CS model with random meeting rates, the measure of supermodularity adjusted for meeting rates coincides with the CS measure of supermodularity.*

Proof. The result follows from plugging the values from Equation (4) into Equation (3); the number of meetings cancel out leaving $SM^{CS}_{t,t'} = SM_{t,t'} - \log(1) = SM_{t,t'}$. □

[8] It is always true that the ratio of the average number of women a man meets to the average number of men a woman meets is equal to $\frac{M}{N}$, but in other models, the number is not the same across types of men or across types of women.

4.2 Model 1: Increased probability of meeting own type

Since we are particularly interested in assortative matching, we want to allow for the possibility that individuals are more likely to meet potential partners of their own type (e.g. education level). Let γ be the additional likelihood of meeting someone of the same type. If we use

$$\gamma_{xy} = \begin{cases} \gamma & x = y, \\ 0 & x \neq y, \end{cases}$$

then we have meeting frequencies

$$a_{xy} = a\frac{m_y}{M}(1+\gamma_{xy}) \cdot \theta \qquad\qquad b_{xy} = b\frac{n_x}{N}(1+\gamma_{xy}) \cdot \theta, \tag{5}$$

where $\theta = \left(1 + \gamma \sum_x \frac{m_x n_x}{NM}\right)^{-1}$ is a multiplier to keep the average number of women a man meets equal to a.[9]

Proposition 4. *In the CS model with increased probability of meeting own type,*

(i) the total systematic surplus is related to the CS surplus by

$$\phi_{xy}^{CS} - \phi_{xy} = \log\left(\theta^2 ab\right) + \log\left(\frac{m_y}{M}\right) + \log\left(\frac{n_x}{N}\right) + 2\log\left(1+\gamma\right) \cdot 1\{x = y\}. \tag{6}$$

(ii) the measure of supermodularity adjusted for meeting rates is related to the CS measure of supermodularity by

$$SM_{t',t} = SM_{t',t}^{CS} - 4\log\left(1+\gamma\right).$$

In particular, if $\gamma = 0$ the two measures coincide. If γ is positive, then the CS measure of supermodularity is an overestimate.

Proof. These results follow immediately from plugging the values from Equation (5) into Proposition 2 and Equation (3), using $x = y = t$ and $x' = y' = t'$ □

In Equation (6), the first three terms come from the random matching. These terms may cause variation in ϕ_{xy}^{CS} over time even if the fundamental surplus is unchanged, because the population counts m_x and n_y can vary over time (and θ depends on these counts). However, since they do not effect the surplus specific to a given pair, they net out when estimating the supermodularity. The last term in Equation (6) only applies for matches between the same type. It therefore affects the estimated supermodularity of the match surplus. Proposition 4 shows that if γ is positive and the econometrician assumes it is zero, that will lead to an overestimate of the supermodularity of the matching surplus. However, if γ has not changed overtime, it cannot explain the changes overtime in the CS measure of supermodularity because it does not interact with the population counts that vary over time.

An informal explanation of the increasing educational assortativeness in marriage matching is that college educated men are more likely to meet college educated women since more women are going to college. However, unless there has also been an *increase in the relative probability* of meeting a given college educated woman verse a given woman without a college education, the model shows this simple explanation is insufficient.

[9] The fact that a and b do not vary by type is again a result of the adding up constraint.

4.3 Model 2: Two ways of meeting

An extension of the idea that more college educated men are meeting more college educated women in college is that they then have less reason to meet women other places. Say that a man i meets γ people from a population that are all his type – e.g. at school or work – and $\frac{m_{x_i}}{m_{x_i}+n_{x_i}}$ of them are female. He then meets the rest of the a total women in proportion to their presence in the general population – e.g. at a bar or at the DMV. The fraction he meets from the general population is

$$q_x = 1 - \frac{\gamma}{a}\frac{m_x}{n_x + m_x}.$$

Similarly, women meet a total of b men; $\gamma\frac{m_{x_i}}{m_{x_i}+n_{x_i}}$ men from only their own type and fraction

$$r_y = 1 - \frac{\gamma}{b}\frac{n_y}{n_y + m_y}$$

from the general population.

Those from the general population are met in proportion to their presence in the population weighted by the number of people they are looking to meet. This gives meeting frequencies

$$a_{xy} = aq_x\frac{m_y r_y}{\sum_{y'} m_{y'} r_{y'}} \qquad b_{xy} = br_y\frac{n_x q_x}{\sum_{x'} n_{x'} q_{x'}}$$

when $x \neq y$. The first term aq_x is the remaining number of women a man of type x wants to meet in the general population. The fraction is the number of men that women of type y are looking to meet, $bm_y r_y$, divided by the number of men that all women are looking to meet, $b\sum_{y'} m_{y'} r_{y'}$.

When $x = y$,

$$a_{xx} = a\left(q_x\frac{m_x r_x}{\sum_{y'} m_{y'} r_{y'}} + \frac{\gamma}{a}\frac{m_x}{n_x + m_x}\right) \qquad b_{xx} = b\left(r_x\frac{n_x q_x}{\sum_{x'} n_{x'} q_{x'}} + \frac{\gamma}{b}\frac{m_y}{n_y + m_y}\right)$$

$$= aq_x\frac{m_x r_x}{\sum_{y'} m_{y'} r_{y'}}\left(1 + \frac{\gamma}{a}\frac{\sum_{y'} m_{y'} r_{y'}}{(n_x + m_x)q_x r_x}\right) \qquad = br_x\frac{n_x q_x}{\sum_{x'} n_{x'} q_{x'}}\left(1 + \frac{\gamma}{b}\frac{\sum_{x'} n_{x'} q_{x'}}{(n_x + m_x)q_x r_x}\right).$$

A man of type x meets $\gamma\frac{m_x}{n_x+m_x}$ women of his own type at work, but also meets $aq_x\frac{m_x r_x}{\sum_{y'} m_{y'} r_{y'}}$ of them in the general population.[10]

When calculating supermodularity, the terms of the form $aq_x\frac{m_y r_y}{\sum_{y'} m_{y'} r_{y'}}$ will cancel, but the ones in parentheses will not; therefore, there is an interaction between the meeting rate $\frac{\gamma}{b}$ and population counts. Let $\theta = \sum_x n_x q_x = b/a\sum_y m_y r_y$, which captures the overall amount of meeting in the general population. We obtain the following result

[10] The meeting frequencies satisfy the adding up constraint $a_{xy}n_x = b_{xy}m_y$ because

$$\frac{1}{a}\sum_y m_y r_y = \frac{N}{M}\frac{1}{b}\left(M - \frac{\gamma}{a}\sum_x \frac{m_x n_x}{m_x + n_x}\right) = \frac{1}{b}\left(N - \frac{\gamma}{a\frac{M}{N}}\sum_x \frac{m_x n_x}{m_x + n_x}\right) = \frac{1}{b}\sum_x n_x q_x.$$

Proposition 5. *Let $x = y = t$ and $x' = y' = t'$. In the CS model with two ways of meeting, the measure of supermodularity adjusted for meeting rates is related to the CS measure of supermodularity by*

$$SM_{t't',tt} = SM^{CS}_{t't',tt} - 2\log\left(\left(1 + \frac{\gamma}{b}\frac{\theta}{(n_t + m_t)q_t r_t}\right)\left(1 + \frac{\gamma}{b}\frac{\theta}{(n_{t'} + m_{t'})q_{t'} r_{t'}}\right)\right). \quad (7)$$

Proof. This result is once again a particular case of Equation (3). □

As in the previous model, the CS measure of supermodularity is an overestimate because it combines the supermodularity of the match surplus and the effect of meeting rates.

Population counts have both a direct effect and an indirect effect (via θ) on the wedge between the CS measure of supermodularity and the measure that accounts for meeting rates. For the direct effect, the intuition is that when there are few women of a given type, they mostly meet partners of the same type; as the number of women of that type grows they meet more partners of other types (though they still disproportionately meet with their own type). The decrease in the extent to which women disproportionately meet men of their own type causes a decrease in the extent to which they disproportionately match to men of their own type. Therefore, moving a woman to a group with fewer women decreases the wedge between the two measures of supermodularity. (See Appendix B for details.)

The wedge between the CS measure of supermodularity and the measure that accounts for meeting rates is increasing in θ, which depends on the population counts. The effect of shifting a woman from t to t' on θ is

$$d\theta = -\frac{\gamma}{a}\left(\left(\frac{n_{t'}}{n_{t'} + m_{t'}}\right)^2 dm_{t'} + \left(\frac{n_t}{n_t + m_t}\right)^2 dm_t\right),$$

so moving a woman between t and t' decreases θ whenever

$$\left(\left(\frac{n_t}{n_t + m_t}\right)^2(-1) + \left(\frac{n_{t'}}{n_{t'} + m_{t'}}\right)^2(1)\right) < 0$$

$$\left(\frac{n_t}{n_t + m_t}\right) < \left(\frac{n_{t'}}{n_{t'} + m_{t'}}\right) \Rightarrow \left(\frac{m_t}{n_t + m_t}\right) > \left(\frac{m_{t'}}{n_{t'} + m_{t'}}\right)$$

The indirect effect of shifting a woman from t to t' depends on the relative gender shares of the two types. Making the gender shares within types more balanced indirectly decreases the wedge between the two measures of supermodularity. If there are the same number of men of both types,[11] then the direct effect and the indirect effect via the amount of searching for partners in the general population (θ) work in the same direction: both indicate that shifting a woman from a type that has more women to a type that has fewer will decrease the wedge between the two measures of supermodularity. Formally, if there are the same number of men of types t' and t and more women of type t than of type t', moving a woman from t to t' will decrease $SM^{CS}_{t',t} - SM_{t',t}$.

Although this is mostly a theoretically-minded paper, we take advantage of Model 2 to illustrate the applicability of our approach. We propose to follow the steps of **?**. Estimation

[11] This is a sufficient, but not necessary condition.

of the parameters of interest only requires a typical cross-sectional dataset (e.g. with multiple cohorts, indexed by c) in which we observe marriage patterns and population supplies for types (e.g. education levels). We have

$$SM^{CS}_{tt,t't',c} \equiv \log\left(\frac{\mu_{tt}\mu_{t't'}}{\mu_{t't}\mu_{tt'}}\right) = SM_{tt,t't',c} + 2\log\left(1 + \frac{\gamma_c}{b_c}\frac{\theta_c}{(n_{t,c}+m_{t,c})q_{t,c}r_{t,c}}\right) + 2\log\left(1 + \frac{\gamma_c}{b_c}\frac{\theta_c}{(n_{t',c}+m_{t',c})q_{t',c}r_{t',c}}\right).$$

where the left-hand side is observed in the data. As in **?**, the first term on the right-hand side can be parameterized as $SM_{tt,t't',c} = \delta_{tt'} + \delta_{tt'} \times c$, where the $\delta_{tt'}$ are dummy variables for each possible pair of types (so that $\delta_{tt'} \times c$ are pair (of types)-specific linear time trends). The second and third terms are non-linear functions of γ_c/b_c, which is identified from variations in the population sizes of different types. This identification is clearly tied to the parametric assumptions; if data on the number of acquaintances (potential partners) between types were also available, some of the structural assumptions could be relaxed.

5 Conclusion

We adapt the Chow and Siow matching model to allow for potential partners to meet each other at different rates. We show that both random meeting and meeting where one is more likely to meet one's own type change the relationship between the match surplus function and the equilibrium matching. Having a higher probability of meeting one's own type creates a wedge between our measure of supermodularity and the CS measure, but not in a way that interacts with population counts; therefore it cannot explain changes over time in the assortativeness estimated from the CS model. We develop an alternative model where meeting additional potential partners of one's own type makes a person seek fewer potential partners of other types. In this model the meeting rates interact with population counts and can lead to changes in the wedge over time; it predicts that the wedge will generally decrease when the ratios within genders become more equal. We briefly discuss the applicability of our setting, arguing that the parameters of interest can be estimated even with limited data. New data sources, such as dating websites (see Hitsch et al (2010) for an early contribution) or survey data on individuals' set of acquaintances, open promising avenues for future research.

Appendix

A Nested Logit

Let the correlation of the random utility shocks for the subtypes within each type be ρ; if $\rho = 1$ then all the sub-types within a type are equivalent and if $\rho = 0$ then the random component of utility is as different across sub-types within a type are as across sub-types of different types. In this case the match probabilities for men satisfy

$$\frac{\mu_{ij}}{\mu_{i0}} = \exp\left(\frac{\alpha_{x_i y_j} - \tau_{x_i y_j}}{1-\rho}\right)\left(\sum_{j' \in c^i_{y_j}} \exp\left(\frac{\alpha_{x_i y_j} - \tau_{x_i y'_j}}{1-\rho}\right)\right)^{-\rho}$$
$$= \exp\left(\frac{\alpha_{ij} - \tau_{x_i y_j}}{1-\rho}\right)^{1-\rho} a^{-\rho}_{x_i y_j}.$$

Combining with the equivalent formula for women, we get

$$\mu^2_{ij} = \exp(\alpha_{x_i y_j} + \gamma_{x_i y_j})\mu_{i0}\mu_{0j} a^{-\rho}_{x_i y_j} b^{-\rho}_{x_i y_j}.$$

Aggregating, as done in the text, gives

$$\mu_{xy} = \left(a^{1-\rho}_{xy} b^{1-\rho}_{xy} \exp(\alpha_{xy} + \gamma_{xy})\mu_{0y}\mu_{x0}\right)^{\frac{1}{2}}.$$

This gives the surplus formula

$$\phi_{xy} = \log\left(\frac{\mu^2_{xy}}{\mu_{0y}\mu_{x0}}\right) - (1-\rho)\log(a_{xy}b_{xy}) = \phi^{CS}_{xy} - (1-\rho)\log(a_{xy}b_{xy}).$$

B Effect of population counts on θ

The increased assortativeness due to meeting (from Equation (7)) is

$$2\log\left(1 + \frac{\gamma}{b}\frac{\theta}{f(n_t, m_t)}\right) + 2\log\left(1 + \frac{\gamma}{b}\frac{\theta}{f(n_{t'}, m_{t'})}\right)$$

where $f(n_t, m_t) = (n_t + m_t)q_t r_t > 0$.

Consider moving a woman from t to t'. This will decrease the assortativeness whenever

$$\frac{-f_2(n_t, m_t)}{f^2(n_t, m_t) + \frac{\theta\gamma}{b} f(n_t, m_t)}(-1) + \frac{-f_2(n_{t'}, m_{t'})}{f^2(n_{t'}, m_{t'}) + \frac{\theta\gamma}{b} f(n_{t'}, m_{t'})}(1) < 0,$$

that is when

$$\frac{f_2(n_{t'}, m_{t'})}{f^2(n_{t'}, m_{t'}) + \frac{\theta\gamma}{b} f(n_{t'}, m_{t'})} > \frac{f_2(n_t, m_t)}{f^2(n_t, m_t) + \frac{\theta\gamma}{b} f(n_t, m_t)}. \tag{8}$$

Expanding $f(\cdot, \cdot)$ (and recalling that $aN = bM$), we have

$$f = (n_t + m_t) - \frac{\gamma}{b} n_t - \frac{\gamma}{b}\frac{N}{M} m_t + \frac{N}{M}\left(\frac{\gamma}{b}\right)^2 \frac{n_t m_t}{n_t + m_t},$$

so

$$f_2 = 1 - \frac{\gamma}{b}\frac{N}{M} + \frac{N}{M}\left(\frac{\gamma}{b}\frac{n_t}{n_t + m_t}\right)^2 \qquad > 0,$$
$$f_{22} = -2\frac{N}{M}\left(\frac{\gamma}{b}\right)^2 \frac{n_t^2}{(m_t + n_t)^3} \qquad < 0.$$

The function f is increasing in the number of women m, and the derivative of f with respect to m is decreasing in m. Therefore, if $n_t = n_{t'}$, moving a woman from t to t' decreases assortativeness (Equation (8) holds) whenever $m_t > m_{t'}$. That is, assortativeness will decrease as we split women more evenly across the two groups.

References

Belot M, Francesconi M (2013) Dating preferences and meeting opportunities in mate choice decisions. Journal of Human Resources 48(2):474–508

Bruze G (2011) Marriage choices of movie stars: Does spouse's education matter? Journal of Human Capital 5(1):1–28

Choo E, Siow A (2006) Who marries whom and why. Journal of Political Economy 114(1):175–201

Dupuy A, Galichon A (2014) Personality traits and the marriage market. Journal of Political Economy 122(6):1271–1319

Eika L, Mogstad M, Zafar B (2017) Educational assortative mating and household income inequality. Working Paper

Galichon A, Kominers SD, Weber S (2017) Costly concessions: An empirical framework for matching with imperfectly transferable utility. Working Paper URL https://ssrn.com/abstract=2535394

Gihleb R, Lang K (2016) Educational homogamy and assortative mating have not increased. Working Paper URL http://www.nber.org/papers/w22927

Goussé M, Jacquemet N, Robin JM (2017) Household labour supply and the marriage market in the UK, 1991-2008. Labour Economics 46:131–149

Greenwood J, Guner N, Kocharkov G, Santos C (2014) Marry your like: Assortative mating and income inequality. American Economic Review 104(5):348–353

Hitsch GJ, Hortasu A, Ariely D (2010) Matching and sorting in online dating. American Economic Review 100(1):130–63

Kalmijn M (1991) Status homogamy in the United States. American Journal of Sociology 97(2):496–523

Lu X, McAfee RP (1996) Matching and expectations in a market with heterogeneous agents. Advances in applied microeconomics 6:121–156

Menzel K (2015) Large matching markets as two-sided demand systems. Econometrica 83(3):897–941

Mourifié I, Siow A (2015) Cohabitation versus marriage: Marriage matching with peer effects. Working Paper

Rosenfeld MJ, Thomas RJ (2012) Searching for a mate: The rise of the internet as a social intermediary. American Sociological Review 77(4):523–547

Schwartz CR, Mare RD (2005) Trends in educational assortative marriage from 1940 to 2003. Demography 42(4):621–646

Shapley LS, Shubik M (1971) The assignment game I: The core. International Journal of Game Theory 1(1):111–130

Shimer R, Smith L (2000) Assortative matching and search. Econometrica 68:343–370